EXERCÍCIO 2

Calcule as primitivas abaixo:

a) $\int \frac{1}{x}\ln(2x)dx$

b) $\int 2tg^3(2x)\sec^2(2x)dx$

c) $\int \frac{-2x}{\sqrt{2-x^2}}dx$

d) $\int \frac{-2x}{-x^2+20}dx$

e) $\int \frac{3e^{3x}}{1+e^{6x}}dx$

f) $\int \frac{3e^{3x}}{(1+e^{3x})^3}dx$

g) $\int \frac{1}{\sqrt{1-x^2}}dx$

h) $\int \frac{5}{\sqrt{1-(5x)^2}}dx$

i) $\int \frac{-50x}{\sqrt{1-(5x)^2}}dx$

j) $\int \cos(\sqrt{x})\frac{1}{2\sqrt{x}}dx$

k) $\int -\frac{1}{2\sqrt{x}}\cos^5(\sqrt{x})sen\sqrt{x}\ \ dx$

l) $\int \frac{4^x+6^x}{2^x}dx$

m) $\int 5e^{5x}+6^x+3\cos(3x)dx$

n) $\int -tg(x)dx$

o) $\int -\sqrt{\cos(x)}sen(x)dx$

p) $\int \frac{-2sen(x)\cos(x)}{\sqrt{1-(sen(x))^2}}dx$

EXERCÍCIO 3

Calcule as seguintes primitivas

a) $\int 3x^4dx$

b) $\int \left(\frac{3}{x}+7\right)dx$

c) $\int 2x^3\sqrt{x}dx$

d) $\int \left(e^{2x}+4^x-\cos(2x)\right)dx$

e) $\int \left(3x+4x\sqrt{x}+\frac{1}{\sqrt{2x}}\right)dx$

f) $\int \frac{\left(x^3-2\sqrt[3]{x}-1+2x\sqrt{x}\right)}{5x}dx$

EXERCÍCIO 4

Calcule as seguintes primitivas

a) $\int (5x+2)^2 dx$

b) $\int \frac{x}{3x^2+1} dx$

c) $\int \frac{x-1}{x^2-2x} dx$

d) $\int tg(x) dx$

e) $\int x(3x^2-5)^5 dx$

f) $\int e^{2x+1} dx$

g) $\int \frac{3x}{x^2+4} dx$

h) $\int \frac{e^{2x}}{\sqrt{e^{2x}+3}} dx$

i) $\int \frac{\ln x}{x} dx, \quad x \geq 2$

j) $\int \frac{2x}{\cos^2(5x^2)} dx$

k) $\int \frac{x}{\sqrt{5-x^2}} dx$

l) $\int \cot g^2(x) dx$

m) $\int \frac{4^x+6^x}{2^{x-1}} dx$

EXERCÍCIO 5

Calcule as seguintes primitivas:

a) $\int \sqrt[8]{x^6} \ dx$

b) $\int sen\ x\ \cos^4 x\ dx$

c) $\int \frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}} dx$

d) $\int \frac{\cos(x)}{sen(x)} dx$

e) $\int \frac{6x^5}{x^6} dx$

f) $\int \frac{6}{6x-8} \ dx$

g) $\int \frac{\frac{1}{x}}{\ln x} \ dx, \quad x > 0$

h) $\int \frac{e^{2x}+2e^{-x}}{e^{2x}+6xe^{-x}} dx$

i) $\int 3sen\ (3x)\ \ dx$

j) $\int 5\sec^2\ (5x)\ \ dx$

k) $\int e^{sen(x)} \cos(x)\ dx$

l) $\int e^{\tan g(x)} \sec^2(x)\ dx$

m) $\int 7xe^{x^2} dx$

n) $\int \frac{5}{1+(5x)^2}\ dx$

o) $\int \frac{5x}{1+(5x)^2}\ dx$

p) $\int \frac{\cos(x)}{1+(sen(x))^2}\ dx$

q) $\int \frac{x^3}{x^8+5} dx$

r) $\int \frac{e^x}{9+25e^{2x}} dx$

s) $\int \frac{e^x}{\sqrt{1-e^{2x}}} dx$

t) $\int \frac{e^x}{\sqrt{1-e^x}} dx$

u) $\int \frac{2x}{\sqrt{1-(2x)^2}}\ dx$

v) $\int \frac{2}{\sqrt{1-(2x)^2}}\ dx$

x) $\int \frac{senx \cos x}{\sqrt{2-sen^4 x}} dx$

y) $\int \frac{e^x}{\sqrt{1-e^{2x}}} dx$

z) $\int \frac{8x}{\sqrt{3-x^2}} dx$

EXERCÍCIO 6

Calcule as seguintes primitivas:

a) $\int \frac{1}{\sqrt{4-x^2}} dx$

b) $\int \frac{sen(x)}{\sqrt{1-\cos^2(x)}} dx$

c) $\int \frac{2x}{\sqrt{1-x^4}} dx$

d) $\int \frac{3x^2}{\sqrt{1-x^6}} dx$

e) $\int \frac{\frac{4}{x}}{\sqrt{4-\ln^2(x)}} dx$

f) $\int \frac{-2sen(x)\cos x}{\sqrt{1-\cos^2(x)}} dx$

g) $\int \frac{-4x^3}{\sqrt{7-x^4}} dx$

h) $\int \frac{1}{4+x^2} dx$

i) $\int \frac{2x}{4+x^2} dx$

j) $\int \frac{x^4}{x^{10}+4} dx$

k) $\int \frac{12x^7}{x^8+5}dx$

l) $\int \frac{e^x}{16+36\left(e^x\right)^2}dx$

m) $\int \frac{2}{\sqrt{3-(2x)^2}}dx$

n) $\int \frac{\cos x}{\sqrt{3-4sen^2(x)}}dx$

o) $\int \frac{8}{\sqrt{3-x^2}}dx$

p) $\int \frac{x}{\sqrt{4-9x^4}}dx$

EXERCÍCIO 7

Calcule as seguintes primitivas:

a) $\int \frac{arctg\left(\frac{x}{2}\right)}{4+x^2}dx$

b) $\int \frac{1}{\sqrt{\left(1+x^2\right)\ln\left(x+\sqrt{1+x^2}\right)}}dx$

c) $\int \frac{\cos\left(arcsen\left(\frac{x}{2}\right)\right)}{\sqrt{4-x^2}}dx$

d) $\int \frac{e^{arctgx}+x\ln\left(1+x^2\right)+1}{1+x^2}dx$

e) $\int \frac{a^{5x}-5}{\sqrt{a^x}}dx$

f) $\int \frac{x+arcsen(2x)}{\sqrt{1-4x^2}}dx$

g) $\int \sqrt{\frac{\ln\left(x+\sqrt{1+x^2}\right)}{1+x^2}}dx$

PROPOSTA DE RESOLUÇÃO

EXERCÍCIO 2

a) $\int \frac{1}{x}\ln(2x)dx$ *(Estamos perante* $\int f^{p} f'$*)*

$$\int \underbrace{\frac{1}{x}}_{f'}\underbrace{(\ln(2x))^1}_{f}dx = \frac{(\ln(2x))^{1+1}}{1+1}+C,\quad C\in\Re$$

$$\int \frac{1}{x}\ln(2x)dx = \frac{(\ln(2x))^2}{2}+C,\quad C\in\Re$$

b) $\int 2tg^3(2x)\sec^2(2x)dx$ *(Estamos perante* $\int f^{p} f'$*)*

$$\int 2tg^3(2x)\sec^2(2x)dx = \int \left(\underbrace{tg\ (2x)}_{f}\right)^{\overbrace{3}^{p}}\underbrace{\sec^2(2x)2}_{f'}dx = \frac{tg^{3+1}(2x)}{3+1}+C,\quad C\in\Re$$

$$\int 2tg^3(2x)\sec^2(2x)dx = \frac{tg^4(2x)}{4}+C,\quad C\in\Re$$

c) $\int \frac{-2x}{\sqrt{2-x^2}}dx$ *(Estamos perante* $\int f^{p} f'$*)*

$$\int \frac{-2x}{\sqrt{2-x^2}}dx = \int \overbrace{-2x}^{f'}\left(\underbrace{2-x^2}_{f}\right)^{\overbrace{\frac{-1}{2}}^{p}}dx = \frac{\left(2-x^2\right)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+C,\quad C\in\Re$$

$$\int \frac{-2x}{\sqrt{2-x^2}}dx = 2\sqrt{2-x^2}+C,\quad C\in\Re$$

d) $\int \frac{-2x}{-x^2+20}dx$ *(Estamos perante* $\int \frac{f'}{f}$*)*

$$\int \frac{\overbrace{-2x}^{f'}}{\underbrace{-x^2+20}_{f}}dx = \ln\left|-x^2+20\right| + C, \quad C \in \Re$$

e) $\int \frac{3e^{3x}}{1+e^{6x}}dx$ *(Estamos perante* $\int \frac{f'}{1+f^2}$ *)*

$$\int \frac{3e^{3x}}{1+e^{6x}}dx = \int \frac{3e^{3x}}{1+(e^{3x})^2}dx = arctg(e^{3x}) + C, \quad C \in \Re$$

f) $\int \frac{3e^{3x}}{\left(1+e^{3x}\right)^3}dx$ *(Estamos perante* $\int f^p f'$ *)*

$$\int \frac{3e^{3x}}{\left(1+e^{3x}\right)^3}dx = \int 3e^{3x}\left(1+e^{3x}\right)^{-3}dx = \frac{\left(1+e^{3x}\right)^{-3+1}}{-3+1} + C, \quad C \in \Re$$

$$\int \frac{3e^{3x}}{\left(1+e^{3x}\right)^3}dx = \frac{\left(1+e^{3x}\right)^{-2}}{-2} + C, \quad C \in \Re$$

g) $\int \frac{1}{\sqrt{1-x^2}}dx$ *(Estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{\overbrace{1}^{f'}}{\sqrt{1-\left(\underbrace{x}_{f}\right)^2}}dx = arcsen(x) + C, \quad C \in \Re$$

h) $\int \frac{5}{\sqrt{1-(5x)^2}}dx$ *(Estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{\overbrace{5}^{f'}}{\sqrt{1-\left(\underbrace{5x}_{f}\right)^2}}dx = arcsen(5x) + C, \quad C \in \Re$$

i) $\int \frac{-50x}{\sqrt{1-(5x)^2}}dx$ *(Estamos perante* $\int f^p f'$ *)*

$$\int \frac{-50x}{\sqrt{1-(5x)^2}}dx = \int \underbrace{-50x}_{f'}\Bigg(\underbrace{1-(5x)^2}_{f}\Bigg)^{\overbrace{\frac{-1}{2}}^{p}} = \frac{\left(1-(5x)^2\right)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + C, C \in \Re$$

$$\int \frac{-50x}{\sqrt{1-(5x)^2}}dx = 2\sqrt{1-(5x)^2} + C, \quad C \in \Re$$

j) $\int \cos(\sqrt{x})\frac{1}{2\sqrt{x}}dx$ *(Estamos perante* $\int \cos f \quad f'$*)*

$$\int \cos(\underbrace{\sqrt{x}}_{f})\overbrace{\frac{1}{2\sqrt{x}}}^{f'}dx = sen\left(\sqrt{x}\right) + C, \quad C \in \Re$$

k) $\int -\frac{1}{2\sqrt{x}}\cos^5(\sqrt{x})sen\sqrt{x} \quad dx$ *(Estamos perante* $\int f^{\,p} f'$*)*

$$\int -\frac{1}{2\sqrt{x}}\cos^5(\sqrt{x})sen\sqrt{x} \quad dx = \int \underbrace{\cos^{\overbrace{5}^{p}}(\sqrt{x})}_{f} \underbrace{sen\sqrt{x}\left(-\frac{1}{2\sqrt{x}}\right)}_{f'}dx = \frac{\cos^6(\sqrt{x})}{6} + C, \quad C \in \Re$$

l) $\int \frac{4^x+6^x}{2^x}dx = \int \frac{4^x}{2^x}dx + \int \frac{6^x}{2^x}dx = \int 2^x dx + \int 3^x dx$ *(Estamos perante* $\int a^f f'$*)*

$$\int 2^x dx + \int 3^x dx = \frac{2^x}{\ln 2} + \frac{3^x}{\ln 3} + C, C \in \Re$$

m) $\int 5e^{5x} + 6^x + 3\cos(3x)dx$ *(Estamos perante* $\int e^f f'$, $\int a^f f'$ *e* $\int \cos f \quad f'$*)*

$$\int 5e^{5x} + 6^x + 3\cos(3x)dx = \int 5e^{5x}dx + \int 6^x dx + \int 3\cos(3x)dx,$$
$$= 5e^{5x} + \frac{6^x}{\ln 6} + sen(3x) + C, C \in \Re$$

n) $\int -tg(x)dx$ *(Estamos perante* $\int \frac{f'}{f}$*)*

$$\int -tg(x)dx = \int \frac{\overbrace{-sen(x)}^{f'}}{\underbrace{\cos(x)}_{f}}dx = \ln|\cos(x)| + C, C \in \Re$$

o) $\int -\sqrt{\cos(x)}sen(x)dx$ *(Estamos perante* $\int f^{p} f'$*)*

$$\int -\sqrt{\cos(x)}sen(x)dx = \int \underbrace{(\cos(x))}_{f}^{\overbrace{\frac{1}{2}}^{p}}\left(\underbrace{-sen(x)}_{f'}\right)$$

p) $\int \frac{-2sen(x)\cos(x)}{\sqrt{1-(sen(x))^2}}dx$ *(Estamos perante* $\int f^{p} f'$*)*

$$\int \underbrace{-2sen(x)\cos(x)}_{f'}\left(\underbrace{1-(sen(x))^2}_{f}\right)^{\overbrace{\frac{-1}{2}}^{p}} dx = \frac{\left(1-(sen(x))^2\right)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + C, \quad C \in \Re$$

$$= 2\left(1-(sen(x))^2\right)^{\frac{1}{2}} + C, \quad C \in \Re$$

$$= 2\sqrt{1-(sen(x))^2} + C, \quad C \in \Re$$

EXERCÍCIO 3

a) $\int 3x^4 dx = 3\int x^4 dx$. *(Estamos perante* $\int f^{p} f'$*)*

$$= 3\int \underbrace{x}_{f}^{\overbrace{4}^{p}} . \underbrace{1}_{f'} dx$$

$$= 3\frac{x^{4+1}}{4+1} + C, \quad C \in \Re$$

$$= 3\frac{x^5}{5} + C, \quad C \in \Re$$

b) $\int \left(\frac{3}{x} + 7\right)dx = \int \frac{3}{x}dx + \int 7dx$

$= 3\int \frac{1}{x}dx + \int 7dx$ *(Estamos perante* $\int \frac{f'}{f}$ *e perante* $\int c$ *)*

$= 3\ln|x| + 7x + C, \quad C \in \Re$

c) $\int 2x^3\sqrt{x}dx = 2\int x^{\frac{7}{2}}dx$ *(Estamos perante* $\int f^p f'$ *)*

$= 2\int \underbrace{x}_{f}^{\overbrace{\frac{7}{2}}^{p}} . \underbrace{1}_{f'} dx$

$= 2\frac{x^{\frac{9}{2}}}{\frac{9}{2}} + C, \quad C \in \Re$

$= \frac{1}{9}x^{\frac{9}{2}} + C, \quad C \in \Re$

$= \frac{1}{9}x^4\sqrt{x} + C, \quad C \in \Re$

d) $\int \left(e^{2x} + 4^x - \cos(2x)\right)dx = \int e^{2x}dx + \int 4^x dx - \int \cos(2x)dx$ *(Com pequenos ajustes, estamos perante* $\int e^f f', \int a^f f', \int \cos f\, f'$ *respectivamente). Vamos multiplicar e dividir a primeira e a terceira primitivas por 2, para obter* $\int e^f f'$ *e* $\int \cos f\, f'$ *respectivamente:*

$\int e^{2x}dx + \int 4^x dx - \int \cos(2x)dx = \frac{1}{2}\int \underbrace{2}_{f'} e^{\overbrace{2x}^{f}}dx + \int 4^x dx - \frac{1}{2}\int \underbrace{2}_{f'} \cos\left(\underbrace{2x}_{f}\right)dx$

$= \frac{1}{2}e^{2x} + \frac{4^x}{\ln 4} - \frac{1}{2}sen(2x) + C, \quad C \in \Re$

e) $\int \left(3x + 4x\sqrt{x} + \frac{1}{\sqrt{2x}}\right)dx = 3\int xdx + 4\int x\sqrt{x}dx + \int \frac{1}{\sqrt{2x}}dx$

$= \quad 3\int x dx + 4\int x^{\frac{3}{2}} dx + \int (2x)^{-\frac{1}{2}} dx$ *(Com pequenos ajustes, estamos perante* $\int f^{p} f'$ *). A terceira primitiva terá que ser multiplicada e dividida por 2, para obtermos* $\int f^{p} f'$ *:*

$$3\int x dx + 4\int x^{\frac{3}{2}} dx + \int (2x)^{-\frac{1}{2}} dx = 3\int \underbrace{x}_{f}^{\overbrace{1}^{p}} . \underbrace{1}_{f'} dx + 4\int \underbrace{x}_{f}^{\overbrace{\frac{3}{2}}^{p}} dx + \frac{1}{2}\int \left(\underbrace{2x}_{f} \right)^{\overbrace{-\frac{1}{2}}^{p}} \underbrace{2}_{f'} dx$$

$$= 3\frac{x^{1+1}}{1+1} + 4\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + \frac{1}{2}\frac{(2x)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + C, \quad C \in \Re$$

$$= 3\frac{x^{2}}{2} + 4\frac{x^{\frac{5}{2}}}{\frac{5}{2}} + (2x)^{\frac{1}{2}} + C, \quad C \in \Re$$

$$= 3\frac{x^{2}}{2} + \frac{8}{5}x^{\frac{5}{2}} + (2x)^{\frac{1}{2}} + C, \quad C \in \Re$$

f) $\int \frac{\left(x^{3} - 2\sqrt[3]{x} - 1 + 2x\sqrt{x}\right)}{5x} dx = \frac{1}{5}\int x^{2} dx - \frac{2}{5}\int \frac{\sqrt[3]{x}}{x} dx - \frac{1}{5}\int \frac{1}{x} dx + \frac{2}{5}\int \sqrt{x} dx$

$\int \frac{\left(x^{3} - 2\sqrt[3]{x} - 1 + 2x\sqrt{x}\right)}{5x} dx = \frac{1}{5}\int x^{2} dx - \frac{2}{5}\int x^{\frac{-2}{3}} dx - \frac{1}{5}\int \frac{1}{x} dx + \frac{2}{5}\int x^{\frac{1}{2}} dx$ *(Estamos perante* $\int f^{p} f'$ *e* $\int \frac{f'}{f}$ *).*

$$= \frac{1}{5}\int \underbrace{x}_{f}^{\overbrace{2}^{p}} . \underbrace{1}_{f'} dx - \frac{2}{5}\int \underbrace{x}_{f}^{\overbrace{\frac{-2}{3}}^{p}} . \underbrace{1}_{f'} dx - \frac{1}{5}\int \frac{\overbrace{1}^{f'}}{\underbrace{x}_{f}} dx + \frac{2}{5}\int \underbrace{x}_{f}^{\overbrace{\frac{1}{2}}^{p}} . \underbrace{1}_{f'} dx$$

$$= \frac{1}{5}\frac{x^{2+1}}{2+1} - \frac{2}{5}\frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} - \frac{1}{5}\ln|x| + \frac{2}{5}\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C, \quad C \in \Re$$

$$= \frac{1}{5}\frac{x^3}{3} - \frac{2}{5}\frac{x^{\frac{1}{3}}}{\frac{1}{3}} - \frac{1}{5}\ln|x| + \frac{2}{5}\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + C, \quad C \in \Re$$

$$= \frac{1}{15}x^3 - \frac{6}{5}x^{\frac{1}{3}} - \frac{1}{5}\ln|x| + \frac{4}{15}x^{\frac{3}{2}} + C, \quad C \in \Re$$

EXERCÍCIO 4

a) $\int (5x+2)^2 dx$ *(Com alguns ajustes, estamos perante* $\int f^{\,p} f'$ *). Vamos multiplicar e dividir a primitiva por 5:*

$$\int (5x+2)^2 dx = \frac{1}{5}\int \Big(\underbrace{5x+2}_{f}\Big)^{\overbrace{2}^{p}} \underbrace{5}_{f'}\, dx$$

$$= \frac{1}{5}\frac{(5x+2)^{2+1}}{2+1} + C, \quad C \in \Re$$

$$= \frac{1}{5}\frac{(5x+2)^3}{3} + C, \quad C \in \Re$$

b) $\int \frac{x}{3x^2+1} dx$ *(Com alguns ajustes, podemos estar perante* $\int \frac{f'}{f}$ *). Vamos multiplicar e dividir a primitiva por 6:*

$$\int \frac{x}{3x^2+1} dx = \frac{1}{6}\int \frac{\overbrace{6x}^{f'}}{\underbrace{3x^2+1}_{f}} dx$$

$$= \frac{1}{6}\ln\left(3x^2+1\right) + C, \quad C \in \Re$$

c) $\int \frac{x-1}{x^2-2x} dx$ *(Com alguns ajustes, podemos estar perante* $\int \frac{f'}{f}$ *). Vamos multiplicar e dividir a primitiva por 2:*

$$\int \frac{x-1}{x^2-2x}dx = \frac{1}{2}\int \frac{2(x-1)}{x^2-2x}dx$$

$$= \frac{1}{2}\int \frac{\overbrace{2x-2}^{f'}}{\underbrace{x^2-2x}_{f}}dx$$

$$= \frac{1}{2}\ln\left|x^2-2x\right| + C, \quad C \in \Re$$

d) $\int tg(x)dx = \int \frac{sen(x)}{\cos(x)}dx$ *(Com alguns ajustes, podemos estar perante* $\int \frac{f'}{f}$ *). Vamos multiplicar e dividir a primitiva por -1:*

$$\int \frac{sen(x)}{\cos(x)}dx = -\int \frac{-sen(x)}{\cos(x)}dx$$

$$= -\ln\left|\cos(x)\right| + C, \quad C \in \Re$$

e) $\int x\left(3x^2-5\right)^5 dx$ *(Com alguns ajustes, estamos perante* $\int f^p f'$ *). Vamos multiplicar e dividir a primitiva por 6:*

$$\int x\left(3x^2-5\right)^5 dx = \frac{1}{6}\int \underbrace{6x}_{f'}\left(\underbrace{3x^2-5}_{f}\right)^5 dx$$

$$= \frac{1}{6}\frac{\left(3x^2-5\right)^{5+1}}{5+1} + C, \quad C \in \Re$$

$$= \frac{1}{6}\frac{\left(3x^2-5\right)^6}{6} + C, \quad C \in \Re$$

f) $\int e^{2x+1}dx$ *(Com alguns ajustes, estamos perante* $\int e^f f'$ *).*

$$\int e^{2x+1}dx = \int e.e^{2x}dx$$

$= e.\int \overbrace{e^{2x}}^{f} dx$ *Vamos multiplicar e dividir a primitiva por 2:*

$$= \frac{e}{2}\int \underbrace{2}_{f'} \overbrace{e^{2x}}^{f} dx$$

$$= \frac{e}{2} e^{2x} + C, \quad C \in \Re$$

g) $\int \frac{3x}{x^2+4} dx$ *(Com alguns ajustes, estamos perante* $\int \frac{f'}{f}$ *). Vamos multiplicar e dividir a primitiva por 2:*

$$3\int \frac{x}{x^2+4} dx = \frac{3}{2}\int \frac{\overbrace{2x}^{f'}}{\underbrace{x^2+4}_{f}} dx$$

$$= \frac{3}{2}\ln\left(x^2+4\right) + C, \quad C \in \Re$$

h) $\int \frac{e^{2x}}{\sqrt{e^{2x}+3}} dx$ *(Com alguns ajustes, estamos perante* $\int f^{p} f'$ *). Vamos multiplicar e dividir a primitiva por 2:*

$$\int \frac{e^{2x}}{\sqrt{e^{2x}+3}} dx = \frac{1}{2}\int \underbrace{2e^{2x}}_{f'} \left(\underbrace{e^{2x}+3}_{f}\right)^{\overbrace{-\frac{1}{2}}^{p}} dx$$

$$= \frac{1}{2} \frac{\left(e^{2x}+3\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C, \quad C \in \Re$$

$$= \left(e^{2x}+3\right)^{\frac{1}{2}} + C, \quad C \in \Re$$

$$= \sqrt{e^{2x}+3} + C, \quad C \in \Re$$

i) $\int \frac{\ln x}{x} dx, \quad x \geq 2$ *(Estamos perante* $\int f^{p} f'$ *).*

$$\int \frac{\ln x}{x} dx = \int \underbrace{\ln^{\overbrace{1}^{p}} x}_{f} \underbrace{\frac{1}{x}}_{f'} dx$$

$$= \frac{\ln^{1+1} x}{1+1} + C, \quad C \in \Re$$

$$= \frac{\ln^{2} x}{2} + C, \quad C \in \Re$$

j) $\int \frac{2x}{\cos^{2}\left(5x^{2}\right)} dx = \int 2x \frac{1}{\cos^{2}\left(5x^{2}\right)} dx$

$= \int (2x) \sec^{2}\left(5x^{2}\right) dx$ *(Com alguns ajustes, estamos perante* $\int f' \sec^{2} f$ *).*

Vamos multiplicar e dividir a primitiva por 5:

$$\int (2x) \sec^{2}\left(5x^{2}\right) dx = \frac{1}{5} \int \underbrace{(10x)}_{f'} \sec^{2}\left(\underbrace{5x^{2}}_{f}\right) dx$$

$$= \frac{1}{5} tg\left(5x^{2}\right) + C, \quad C \in \Re$$

k) $\int \frac{x}{\sqrt{5-x^{2}}} dx$ *(Com alguns ajustes, estamos perante* $\int f^{p} f'$ *). Vamos multiplicar e dividir a primitiva por -2:*

$$\int \frac{x}{\sqrt{5-x^{2}}} dx = -\frac{1}{2} \int \frac{-2x}{\sqrt{5-x^{2}}} dx$$

$$= -\frac{1}{2} \int \overbrace{-2x}^{f'} \left(\underbrace{5-x^{2}}_{f}\right)^{\overbrace{\frac{-1}{2}}^{p}} dx$$

$$= \frac{-1}{2}\frac{\left(5-x^2\right)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1} + C, \quad C \in \Re$$

$$= -\left(5-x^2\right)^{\frac{1}{2}} + C, \quad C \in \Re$$

$$= -\sqrt{\left(5-x^2\right)} + C, \quad C \in \Re$$

l) $\int \cot g^2(x)dx = \int \frac{\cos^2(x)}{sen^2(x)}dx$

$$= \int \frac{1-sen^2(x)}{sen^2(x)}dx$$

$$= \int \frac{1}{sen^2(x)}dx - \int 1dx$$

$= \int \cos ec^2(x)dx - \int 1dx$ *Estamos perante* $\int f' \cos ec^2 f \quad dx - \int c \quad dx$

$$= -\cot g(x) - x + C, \quad C \in \Re$$

m) $\int \frac{4^x + 6^x}{2^{x-1}}dx = \int \frac{4^x + 6^x}{2^x \cdot 2^{-1}}dx$

$$= 2\int \frac{4^x + 6^x}{2^x}dx$$

$$= 2\int \frac{2^x\left(2^x + 3^x\right)}{2^x}dx$$

$$= 2\int \left(2^x + 3^x\right)dx$$

$= 2\int 2^x dx + 2\int 3^x dx$ *(Estamos perante* $\int a^f f'$ *).*

$$= 2\frac{2^x}{\ln 2} + 2\frac{3^x}{\ln 3} + C, \quad C \in \Re$$

$$= 2\left(\frac{2^x}{\ln 2} + \frac{3^x}{\ln 3}\right) + C, \quad C \in \Re$$

EXERCÍCIO 5

a) $\int \sqrt[8]{x^6}\ dx = \int x^{\frac{6}{8}}\ dx$ *. (Estamos perante* $\int f^{\,p} f'$ *)*

$$\int x^{\frac{6}{8}}\ dx = \frac{x^{\frac{6}{8}+1}}{\frac{6}{8}+1} + C,\ \ C \in \Re$$

$$= \frac{x^{\frac{14}{8}}}{\frac{14}{8}} + C,\ \ C \in \Re$$

$$= \frac{8}{14} x^{\frac{14}{8}} + C,\ \ C \in \Re$$

$$= \frac{4}{7} x^{\frac{14}{8}} + C,\ \ C \in \Re$$

b) $\int sen\ x\ \cos^4 x\ dx = -\int - senx\ \ \cos^4 x\ \ dx$ *(Estamos perante* $\int f^{\,p} f'$ *)*

$$= -\frac{\cos^{4+1}(x)}{4+1} + C,\ \ C \in \Re$$

$$= -\frac{\cos^{5}(x)}{5} + C,\ \ C \in \Re$$

c) $\int \frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}} dx$ *(Com alguns ajustes, podemos estar perante* $\int f^{\,p} f'$ *)*

$$\int \frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}} dx = \int \frac{x^4 - x^2 - 2}{x^{\frac{2}{3}}} dx$$

$$= \int \left(x^4 - x^2 - 2\right)\ x^{-\frac{2}{3}} dx$$

$$= \int \left(x^4 x^{-\frac{2}{3}} - x^2 x^{-\frac{2}{3}} - 2x^{-\frac{2}{3}} \right) dx$$

$$= \int \left(x^{\frac{10}{3}} - x^{\frac{4}{3}} - 2x^{-\frac{2}{3}} \right) dx$$

$$= \int x^{\frac{10}{3}} dx + \int \left(-x^{\frac{4}{3}} \right) dx - 2\int \left(x^{-\frac{2}{3}} \right) dx$$ *Estamos agora perante* $\int f^p f'$ *). Então*

$$\int \frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}} dx = \frac{x^{\frac{10}{3}+1}}{\frac{10}{3}+1} + \frac{x^{\frac{4}{3}+1}}{\frac{4}{3}+1} - 2\frac{x^{\frac{-2}{3}+1}}{\frac{-2}{3}+1} + C, \quad C \in \Re$$

$$\int \frac{(x^2+1)(x^2-2)}{\sqrt[3]{x^2}} dx = \frac{x^{\frac{13}{3}}}{\frac{13}{3}} + \frac{x^{\frac{7}{3}}}{\frac{7}{3}} - 2\frac{x^{\frac{1}{3}}}{\frac{1}{3}} + C, \quad C \in \Re$$

$$= \frac{3}{13} x^{\frac{13}{3}} + \frac{3}{7} x^{\frac{7}{3}} - 6x^{\frac{1}{3}} + C, \quad C \in \Re$$

d) $\int \frac{\cos(x)}{sen(x)} dx$ *(Estamos perante* $\int \frac{f'}{f}$ *)*

$$\int \frac{\cos(x)}{sen(x)} dx = \ln|sen(x)| + C, \quad C \in \Re$$

e) $\int \frac{6x^5}{x^6} dx = 6\int \frac{1}{x} dx$ *(Estamos perante* $\int \frac{f'}{f}$ *)*

$$= 6\ln|x| + C, \quad C \in \Re$$

f) $\int \frac{6}{6x-8} dx$ *(Estamos perante* $\int \frac{f'}{f}$ *)*

$$\int \frac{6}{6x-8}dx = \ln|6x-8| + C, \quad C \in \Re$$

g) $\int \frac{\frac{1}{x}}{\ln x}\, dx, \quad x>0$ *(Estamos perante* $\int \frac{f'}{f}$*)*

$$\int \frac{\frac{1}{x}}{\ln x}dx = \ln|\ln x| + C, \quad C \in \Re$$

h) $\int \frac{e^{2x}+2e^{-x}}{e^{2x}+6xe^{-x}}dx$ *(Com alguns ajustes, podemos estar perante* $\int \frac{f'}{f}$*)*

$$\int \frac{e^{2x}+2e^{-x}}{e^{2x}+6xe^{-x}}dx = \int \frac{e^{-x}\left(e^{3x}+2\right)}{e^{-x}\left(e^{3x}+6x\right)}dx$$

$$= \int \frac{\left(e^{3x}+2\right)}{\underbrace{\left(e^{3x}+6x\right)}_{f}}dx$$ *multiplicando e dividindo a primitiva por 3,*

obtemos:

$$\frac{1}{3}\int \frac{3\left(e^{3x}+2\right)}{\underbrace{\left(e^{3x}+6x\right)}_{f}}dx$$

$$= \frac{1}{3}\int \frac{\overbrace{3e^{3x}+6}^{f'}}{\underbrace{\left(e^{3x}+6x\right)}_{f}}dx$$

$$= \frac{1}{3}\ln\left|e^{3x}+6x\right| + C, \quad C \in \Re$$

i) $\int 3sen\ (3x)\ \ dx$ *(Estamos perante* $\int f' \ \ sen(f)$*)*

$$\int 3sen\ (3x)\ \ dx = -\cos(3x) + C, \quad C \in \Re$$

j) $\int 5\sec^2(5x)\ dx$ *(Estamos perante* $\int f'\ \sec^2(f)$*)*

$$\int 5\ \sec^2(5x)dx = \tan g(5x) + C,\quad C \in \Re$$

k) $\int e^{sen(x)}\cos(x)\ dx$ *(Estamos perante* $\int e^f\ f'$*)*

$$\int \underbrace{e^{sen(x)}}_{e^f}\underbrace{\cos(x)}_{f'}dx = e^{sen(x)} + C,\quad C \in \Re$$

l) $\int e^{\tan g(x)}\sec^2(x)\ dx$ *(Estamos perante* $\int e^f\ f'$*)*

$$\int e^{\tan g(x)}\sec^2(x)\ dx = e^{\tan g(x)} + C,\quad C \in \Re$$

m) $\int 7xe^{x^2}dx$ *(Com alguns ajustes, podemos estar perante* $\int e^f\ f'$*). Vejamos:*

$$\int 7xe^{x^2}dx = 7\int xe^{x^2}\ dx$$

Multipliquemos e dividamos a primitiva por 2

$$7\int xe^{x^2}\ dx = \frac{7}{2}\int 2xe^{x^2}\ dx$$

Agora já estamos perante $\int f'\ \ e^f$

$$= \frac{7}{2}e^{x^2} + C,\quad C \in \Re$$

n) $\int \frac{5}{1+(5x)^2}\ dx$ *(Estamos perante* $\int \frac{f'}{1+f^2}$*)*

$$\int \frac{5}{1+(5x)^2}\ dx = arctg(5x) + C,\quad C \in \Re$$

o) $\int \frac{5x}{1+(5x)^2}\,dx$ *(Com alguns ajustes, podemos estar perante* $\int \frac{f'}{f}$ *)*

$\int \frac{5x}{1+(5x)^2}\,dx = \int \frac{5x}{1+25x^2}\,dx$, *se multiplicarmos e dividirmos a primitiva por 10, vamos estar perante* $\int \frac{f'}{f}$. *Vejamos:*

$$\int \frac{5x}{1+25x^2}\,dx = \frac{1}{10}\int \frac{\overbrace{50x}^{f'}}{\underbrace{1+25x^2}_{f}}\,dx$$

$$= \frac{1}{10}\ln\left(1+25x^2\right)+C,\quad C\in\Re$$

p) $\int \frac{\cos(x)}{1+(sen(x))^2}\,dx$ *(Estamos perante* $\int \frac{f'}{1+f^2}$ *)*

$$\int \frac{\cos(x)}{1+(sen(x))^2}\,dx = \int \frac{\cos(x)}{1+sen^2(x)}\,dx$$

$$= arctg\left(sen\ (x)\right)+C,\quad C\in\Re$$

q) $\int \frac{x^3}{x^8+5}dx$ *(Mediante algumas transformações, estamos perante* $\int \frac{f'}{1+f^2}$ *)*

$\int \frac{x^3}{x^8+5}dx = \int \frac{x^3}{5\left(1+\left(\frac{x^4}{\sqrt{5}}\right)^2\right)}dx$ *Multiplicando e dividindo a primitiva por* $\frac{4}{\sqrt{5}}$, *vamos obter:*

$$= \frac{\sqrt{5}}{4}\frac{1}{5}\int \frac{\overbrace{\frac{4}{\sqrt{5}}x^3}^{f'}}{\left(1+\left(\underbrace{\frac{x^4}{\sqrt{5}}}_{f}\right)^2\right)}dx$$

, estando agora perante $\int \frac{f'}{1+f^2}$

Temos então:

$$\int \frac{x^3}{x^8+5}dx = \frac{\sqrt{5}}{20} arctg\left(\frac{x^4}{\sqrt{5}}\right) + C, \quad C \in \Re$$

r) $\int \frac{e^x}{9+25e^{2x}}dx$ *(Com alguns ajustes, podemos estar perante* $\int \frac{f'}{1+f^2}$ *)*

$$\int \frac{e^x}{9+25e^{2x}}dx = \int \frac{e^x}{9\left(1+\left(\underbrace{\frac{5e^x}{3}}_{f}\right)^2\right)}dx$$

$$= \frac{1}{9}\int \frac{e^x}{\left(1+\left(\underbrace{\frac{5e^x}{3}}_{f}\right)^2\right)}dx$$ *Multiplicando e dividindo a primitiva por* $\frac{5}{3}$*, estaremos perante* $\int \frac{f'}{1+f^2}$*. Vejamos*

$$\frac{1}{9}\int \frac{e^x}{\left(1+\left(\underbrace{\frac{5e^x}{3}}_{f}\right)^2\right)}dx = \frac{1}{9}\frac{3}{5}\int \frac{\overbrace{\frac{5}{3}e^x}^{f'}}{\left(1+\left(\underbrace{\frac{5e^x}{3}}_{f}\right)^2\right)}dx$$

$$= \frac{1}{15} arctg\left(\frac{5e^x}{3}\right) + C, \quad C \in \Re$$

s) $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$ *(Estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{\overbrace{e^x}^{f'}}{\sqrt{1-\left(e^x\right)^2}}dx = arcsen\left(e^x\right)+C,\quad C\in\Re$$

t) $\int \frac{e^x}{\sqrt{1-e^x}}dx$ *(Com alguns ajustes, podemos estar perante* $\int f^{\,p} f'$ *)*

$$\int \frac{e^x}{\sqrt{1-e^x}}dx = \int e^x \left(\underbrace{1-e^x}_{f}\right)^{\frac{-1}{2}} dx$$ *, multiplicando e dividindo a primitiva por -1, obtemos:*

$$= -\int \underbrace{-e^x}_{f'}\left(\underbrace{1-e^x}_{f}\right)^{\frac{-1}{2}} dx$$

$$= \frac{-\left(1-e^x\right)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+C,\quad C\in\Re$$

$$= 2\left(-\left(1-e^x\right)^{\frac{1}{2}}\right)+C,\quad C\in\Re$$

$$= -2\sqrt{\left(1-e^x\right)}+C,\quad C\in\Re$$

u) $\int \frac{2x}{\sqrt{1-(2x)^2}}$ dx *(Com alguns ajustes, podemos estar perante* $\int f^{\,p} f'$ *)*

$$\int \frac{2x}{\sqrt{1-(2x)^2}}\ dx = \int \left(1-(2x)^2\right)^{-\frac{1}{2}}\ 2x\ \ dx$$

Se multiplicarmos e dividirmos a primitiva por -4, teremos

$$-\frac{1}{4}\int \underbrace{\left(1-(2x)^2\right)^{-\frac{1}{2}}}_{f^{\,p}}\ \underbrace{(-8x)}_{f'}\ dx$$ *, obtendo assim* $\int f^{\,p} f'$

$$-\frac{1}{4}\int \underbrace{\left(1-(2x)^2\right)^{-\frac{1}{2}}}_{f^{\,p}}\ \underbrace{(-8x)}_{f'}\ dx = -\frac{1}{4}\,\frac{\left(1-(2x)^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C,\quad C\in\Re$$

$$= -\frac{1}{4}\frac{\left(1-(2x)^2\right)^{\frac{1}{2}}}{-\frac{1}{2}+1}+C, \quad C \in \Re$$

$$= -\frac{1}{2}\left(1-(2x)^2\right)^{\frac{1}{2}}+C, \quad C \in \Re$$

$$= -\frac{1}{2}\sqrt{\left(1-(2x)^2\right)}+C, \quad C \in \Re$$

v) $\int \frac{2}{\sqrt{1-(2x)^2}}\,dx$ *(Estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{2}{\sqrt{1-(2x)^2}}\,dx = arcsen(2x)+C, \quad C \in \Re$$

x) $\int \frac{senx\cos x}{\sqrt{2-sen^4 x}}\,dx$ *(mediante algumas transformações estaremos perante* $\int \frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{sen(x)\cos(x)}{\sqrt{2-sen^4(x)}}\,dx = \int \frac{sen(x)\cos(x)}{\sqrt{2\left(1-\left(\frac{sen^2(x)}{\sqrt{2}}\right)^2\right)}}\,dx$$

$$= \int \frac{sen(x)\cos(x)}{\sqrt{2}\sqrt{\left(1-\left(\frac{sen^2(x)}{\sqrt{2}}\right)^2\right)}}\,dx$$ *Multiplicando e dividindo a primitiva por* $\frac{2}{\sqrt{2}}$*, vamos obter:*

$$\int \frac{sen(x)\cos(x)}{\sqrt{2-sen^4(x)}}\,dx = \frac{\sqrt{2}}{2}\int \frac{\frac{2}{\sqrt{2}}sen(x)\cos(x)}{\sqrt{2}\sqrt{\left(1-\left(\frac{sen^2(x)}{\sqrt{2}}\right)^2\right)}}\,dx$$

$$=\frac{\sqrt{2}}{2\sqrt{2}}\int \frac{\overbrace{\frac{2}{\sqrt{2}}sen(x)\cos(x)}^{f'}}{\sqrt{\left(1-\underbrace{\left(\frac{sen^2(x)}{\sqrt{2}}\right)}_{f}^2\right)}}dx$$, *estamos agora então perante*

$\int \frac{f'}{\sqrt{1-f^2}}$*, logo*

$$\int \frac{sen(x)\cos(x)}{\sqrt{2-sen^4(x)}}dx=\frac{1}{2}arcsen\left(\frac{sen^2(x)}{\sqrt{2}}\right)+C,\quad C\in\Re$$

y) $\int \frac{e^x}{\sqrt{1-e^{2x}}}dx$ *(Estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{e^x}{\sqrt{1-e^{2x}}}dx=\int \frac{e^x}{\sqrt{1-\left(e^x\right)^2}}dx$$

$$=arcsen\left(e^x\right)+C,\quad C\in\Re$$

z) $\int \frac{8x}{\sqrt{3-x^2}}dx$ *(Com alguns ajustes, podemos estar perante* $\int f^p f'$ *)*

$$\int \frac{8x}{\sqrt{3-x^2}}dx=8\int x\left(\underbrace{3-x^2}_{f}\right)^{\frac{-1}{2}}dx$$ *Multiplicando e dividindo a primitiva por -2, temos:*

$$8\int x\left(\underbrace{3-x^2}_{f}\right)^{\frac{-1}{2}}dx=\frac{8}{-2}\int -2x\left(\underbrace{3-x^2}_{f}\right)^{\frac{-1}{2}}dx$$

$$=-4\int -2x\left(\underbrace{3-x^2}_{f}\right)^{\frac{-1}{2}}dx$$

$$= -4\frac{\left(\left(3-x^2\right)\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1}+C,\quad C\in\Re$$

$$= -8\left(3-x^2\right)^{\frac{1}{2}}+C,\quad C\in\Re$$

$$= -8\sqrt{3-x^2}+C,\quad C\in\Re$$

EXERCÍCIO 6

a) $\int\frac{1}{\sqrt{4-x^2}}dx$ *(Com algumas alterações, estamos perante* $\int\frac{f'}{\sqrt{1-f^2}}$ *)*

$\int\frac{1}{\sqrt{4\left[1-\left(\frac{x}{2}\right)^2\right]}}dx=\frac{1}{2}\int\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}dx$ *temos agora que multiplicar o numerador*

por $\left(\frac{x}{2}\right)'=\frac{1}{2}$*, para obter* $\int\frac{f'}{\sqrt{1-f^2}}$*.Temos então:*

$\frac{1}{2}\int\frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}dx=\frac{1}{2}\times\frac{1}{\frac{1}{2}}\int\frac{1\times\frac{1}{2}}{\sqrt{1-\left(\frac{x}{2}\right)^2}}dx$ *(multiplicou-se a primitiva por* $\frac{1}{\frac{1}{2}}$*, para não alterar o seu valor, uma vez que se tinha multiplicado por* $\frac{1}{2}$*. Temos então:*

$$\int\frac{1}{\sqrt{4-x^2}}dx=\int\frac{\frac{1}{2}}{\sqrt{1-\left(\frac{x}{2}\right)^2}}dx=arcsen\left(\frac{x}{2}\right)+C,\quad C\in\Re$$

b) $\int\frac{sen(x)}{\sqrt{1-\cos^2(x)}}dx$ *(Com algumas alterações, estamos perante* $\int\frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{sen(x)}{\sqrt{1-\cos^2(x)}}dx = -\int \frac{-sen(x)}{\sqrt{1-\cos^2(x)}}dx$$ *multiplicámos o numerador por -1, e obviamente tivemos que dividir a primitiva por -1 para não alterar o seu valor.*

$$\int \frac{sen(x)}{\sqrt{1-\cos^2(x)}}dx = -\int \frac{\overbrace{-sen(x)}^{f'}}{\sqrt{1-\left(\underbrace{\cos(x)}_{f}\right)^2}}dx == -arcsen(\cos(x))+C,\quad C\in\Re$$

c) $\int \frac{2x}{\sqrt{1-x^4}}dx$ *(Com algumas alterações, estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{2x}{\sqrt{1-x^4}}dx = \int \frac{\overbrace{2x}^{f'}}{\sqrt{1-\left(\underbrace{(x)^2}_{f}\right)^2}}dx \quad \int \frac{2x}{\sqrt{1-x^4}}dx = arcsen\left((x)^2\right)+C,\quad C\in\Re$$

d) $\int \frac{3x^2}{\sqrt{1-x^6}}dx = \int \frac{3x^2}{\sqrt{1-\left(x^3\right)^2}}dx$ *(Estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{3x^2}{\sqrt{1-x^6}}dx = arcsen\left((x)^3\right)+C,\quad C\in\Re$$

e) $\int \frac{\frac{4}{x}}{\sqrt{4-\ln^2(x)}}dx$ *(Com algumas alterações, estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{\frac{4}{x}}{\sqrt{4-\ln^2(x)}}dx = \int \frac{\frac{4}{x}}{\sqrt{4\left(1-\frac{\ln^2(x)}{4}\right)}}dx = \frac{1}{2}\int \frac{\frac{4}{x}}{\sqrt{\left(1-\frac{\ln^2(x)}{4}\right)}}dx = \frac{4}{2}\int \frac{\frac{1}{x}}{\sqrt{\left(1-\left(\underbrace{\frac{\ln\ (x)}{2}}_{f}\right)^2\right)}}dx$$

$$= 2\times\frac{1}{\frac{1}{2}}\int\frac{\frac{1}{x}\times\frac{1}{2}}{\sqrt{\left(1-\left(\underbrace{\frac{\ln(x)}{2}}_{f}\right)^2\right)}}dx = 4\int\frac{\overbrace{\frac{1}{2x}}^{f'}}{\sqrt{\left(1-\left(\underbrace{\frac{\ln(x)}{2}}_{f}\right)^2\right)}}dx = 4arcsen\left(\frac{\ln(x)}{2}\right)+C, C\in\Re$$

f) $\int\frac{-2sen(x)\cos(x)}{\sqrt{1-\cos^2(x)}}dx$ *(Com algumas alterações, estamos perante* $\int f^{p} f'$*)*

$$\int\frac{-2sen(x)\cos(x)}{\sqrt{1-\cos^2(x)}}dx = \int\overbrace{-2sen(x)\cos(x)}^{f'}\left(\underbrace{1-\cos^2(x)}_{f}\right)^{\overbrace{\frac{-1}{2}}^{p}}dx = \frac{\left(1-\cos^2(x)\right)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+C,\quad C\in\Re$$

$$\int\frac{-2sen(x)\cos(x)}{\sqrt{1-\cos^2(x)}}dx = \frac{\left(1-\cos^2(x)\right)^{\frac{1}{2}}}{\frac{1}{2}}+C,\quad C\in\Re = 2\sqrt{1-\cos^2(x)}+C,\quad C\in\Re$$

g) $\int\frac{-4x^3}{\sqrt{7-x^4}}dx$ *(Com algumas alterações, estamos perante* $\int f^{p} f'$*)*

$$\int\frac{-4x^3}{\sqrt{7-x^4}}dx = \int\underbrace{-4x^3}_{f'}\left(\underbrace{7-x^4}_{f}\right)^{\overbrace{\frac{-1}{2}}^{p}}dx = \frac{\left(7-x^4\right)^{\frac{-1}{2}+1}}{\frac{-1}{2}+1}+C,\quad C\in\Re$$

$$\int\frac{-4x^3}{\sqrt{7-x^4}}dx = = \frac{\left(7-x^4\right)^{\frac{1}{2}}}{\frac{1}{2}}+C = 2\sqrt{7-x^4}+C,\quad C\in\Re$$

h) $\int\frac{1}{4+x^2}dx$ *(Com algumas alterações, estamos perante* $\int\frac{f'}{1+f^2}$*)*

$$\int \frac{1}{4+x^2}dx = \int \frac{1}{4\left(1+\frac{x^2}{4}\right)}dx = \frac{1}{4}\int \frac{1}{1+\left(\underbrace{\frac{x}{2}}_{f}\right)^2}dx = \frac{1}{4}\times\frac{1}{\frac{1}{2}}\int \frac{1\times\frac{1}{2}}{1+\left(\underbrace{\frac{x}{2}}_{f}\right)^2}dx = \frac{1}{2}\int \frac{1\times\frac{1}{2}}{1+\left(\underbrace{\frac{x}{2}}_{f}\right)^2}dx$$

$$\int \frac{1}{4+x^2}dx = \frac{1}{2}arctg\left(\frac{x}{2}\right)+C, \quad C \in \Re$$

i) $\int \frac{2x}{4+x^2}dx$ *(Estamos perante* $\int \frac{f'}{f}$*)*

$$\int \frac{2x}{4+x^2}dx = \ln\left|4+x^2\right| + C, \quad C \in \Re$$

j) $\int \frac{x^4}{x^{10}+4}dx$ *(Com algumas alterações, estamos perante* $\int \frac{f'}{1+f^2}$*)*

$$\int \frac{x^4}{4+x^{10}}dx = \int \frac{x^4}{4\left(1+\frac{\left(x^5\right)^2}{4}\right)}dx = \frac{1}{4}\int \frac{x^4}{\left(1+\left(\frac{x^5}{2}\right)^2\right)}dx = \frac{1}{4}\times\frac{2}{5}\int \frac{\overbrace{\frac{5}{2}\times x^4}^{f'}}{1+\left(\underbrace{\frac{x^5}{2}}_{f}\right)^2}dx$$

$$\int \frac{x^4}{4+x^{10}}dx = \frac{1}{10}arctg\left(\frac{x^5}{2}\right)+C, \quad C \in \Re$$

k) $\int \frac{12x^7}{x^8+5}dx$ *(Com algumas alterações, estamos perante* $\int \frac{f'}{f}$*)*

$$\int \frac{12x^7}{x^8+5}dx = 12\int \frac{x^7}{\underbrace{x^8+5}_{f}}dx = \frac{12}{8}\int \frac{\overbrace{8\times x^7}^{f'}}{\underbrace{x^8+5}_{f}}dx = \frac{3}{2}\ln\left|x^8+5\right|+C, \quad C \in \Re$$

l) $\int \frac{e^x}{16+36\left(e^x\right)^2}dx$ *(Com algumas alterações, estamos perante* $\int \frac{f'}{1+f^2}$*)*

$$\int \frac{e^x}{16+36\left(e^x\right)^2}dx = \int \frac{e^x}{16\left(1+\frac{36}{16}\left(e^x\right)^2\right)}dx = \frac{1}{16}\int \frac{e^x}{\left(1+\left(\underbrace{\frac{6}{4}e^x}_{f}\right)^2\right)}dx$$

$$= \frac{1}{16}\times\frac{4}{6}\int \frac{e^x\times\frac{6}{4}}{\left(1+\left(\underbrace{\frac{6}{4}e^x}_{f}\right)^2\right)}dx = \frac{1}{24}arctg\left(\frac{6}{4}e^x\right)+C = \frac{1}{24}arctg\left(\frac{3}{2}e^x\right)+C,\quad C\in\Re$$

m) $\int \frac{2}{\sqrt{3-(2x)^2}}dx$ *(Com algumas alterações, estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$*)*

$$\int \frac{2}{\sqrt{3\left(1-\frac{1}{3}(2x)^2\right)}}dx = \frac{1}{\sqrt{3}}\int \frac{2}{\sqrt{\left(1-\left(\frac{2x}{\sqrt{3}}\right)^2\right)}}dx = \frac{1}{\sqrt{3}}\times\frac{\sqrt{3}}{2}\int \frac{2\times\frac{2}{\sqrt{3}}}{\sqrt{\left(1-\left(\frac{2x}{\sqrt{3}}\right)^2\right)}}dx$$

$$= \frac{1}{2}\int \frac{2\times\frac{2}{\sqrt{3}}}{\sqrt{\left(1-\left(\frac{2x}{\sqrt{3}}\right)^2\right)}}dx = \frac{1}{2}arcsen\left(\frac{2x}{\sqrt{3}}\right)+C,\quad C\in\Re$$

n) $\int \frac{\cos(x)}{\sqrt{3-4sen^2(x)}}dx$ *(Com algumas alterações, estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$*)*

$$\int \frac{\cos(x)}{\sqrt{3-4sen^2(x)}}dx = \frac{1}{\sqrt{3}}\int \frac{\cos(x)}{\sqrt{\left(1-\frac{4sen^2(x)}{3}\right)}}dx = \frac{1}{\sqrt{3}}\int \frac{\cos(x)}{\sqrt{\left(1-\left(\frac{2sen(x)}{\sqrt{3}}\right)^2\right)}}$$

$$= \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} \int \frac{\frac{2}{\sqrt{3}}\cos(x)}{\sqrt{\left(1-\left(\frac{2sen(x)}{\sqrt{3}}\right)^2\right)}} = \frac{1}{2} arcsen\left(\frac{2}{\sqrt{3}} sen(x)\right) + C, \quad C \in \Re$$

o) $\int \frac{8}{\sqrt{3-x^2}} dx$ *(Mediante algumas transformações, estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{8}{\sqrt{3-x^2}} dx = 8\int \frac{1}{\sqrt{3\left(1-\left(\frac{x}{\sqrt{3}}\right)^2\right)}} dx$$

$$= \frac{8}{\sqrt{3}} \int \frac{1}{\sqrt{\left(1-\left(\underbrace{\frac{x}{\sqrt{3}}}_{f}\right)^2\right)}} dx$$ *, multiplicando e dividindo a primitiva por*

$\frac{1}{\sqrt{3}}$*, temos:*

$$\frac{8}{\sqrt{3}}\sqrt{3}\int \frac{\overbrace{\frac{1}{\sqrt{3}}}^{f'}}{\sqrt{\left(1-\left(\underbrace{\frac{x}{\sqrt{3}}}_{f}\right)^2\right)}} dx = 8\int \frac{\overbrace{\frac{1}{\sqrt{3}}}^{f'}}{\sqrt{\left(1-\left(\underbrace{\frac{x}{\sqrt{3}}}_{f}\right)^2\right)}} dx$$

$$= 8arcsen\left(\frac{x}{\sqrt{3}}\right) + C, \quad C \in \Re$$

p) $\int \frac{x}{\sqrt{4-9x^4}} dx$ *(Mediante algumas alterações, estamos perante* $\int \frac{f'}{\sqrt{1-f^2}}$ *)*

$$\int \frac{x}{\sqrt{4-9x^4}}dx = \int \frac{x}{\sqrt{4\left(1-\frac{9x^4}{4}\right)}}dx = \frac{1}{2}\int \frac{x}{\sqrt{\left(1-\frac{9x^4}{4}\right)}}dx = \frac{1}{2}\int \frac{x}{\sqrt{\left(1-\left(\frac{3x^2}{2}\right)^2\right)}}dx$$

$$= \frac{1}{2}\times\frac{1}{3}\int \frac{\overbrace{3x}^{f'}}{\sqrt{\left(1-\left(\underbrace{\frac{3x^2}{2}}_{f}\right)^2\right)}}dx = \frac{1}{6}arcsen\left(\frac{3x^2}{2}\right)+C,\quad C\in\Re$$

EXERCÍCIO 6

a) $\int \frac{arctg\left(\frac{x}{2}\right)}{4+x^2}dx$ *(Com algumas transformações estamos perante* $\int f^{\,p} f'$ *)*

$$\int \frac{arctg\left(\frac{x}{2}\right)}{4+x^2}dx = \int arctg\left(\frac{x}{2}\right)\frac{1}{4\left(1+\left(\frac{x}{2}\right)^2\right)}dx = \frac{1}{4}\int arctg\left(\frac{x}{2}\right)\frac{1}{\left(1+\left(\frac{x}{2}\right)^2\right)}dx =$$

$$= \frac{1}{4}\int arctg\left(\frac{x}{2}\right)\frac{1}{\left(1+\left(\frac{x}{2}\right)^2\right)}dx = \frac{1}{4}\times 2\int arctg\left(\frac{x}{2}\right)\frac{1\times\frac{1}{2}}{\left(1+\left(\frac{x}{2}\right)^2\right)}dx$$

$$= \frac{1}{2}\frac{arctg^2\left(\frac{x}{2}\right)}{2}+C, C\in\Re$$

b) $\int \frac{1}{\sqrt{(1+x^2)\ln\left(x+\sqrt{1+x^2}\right)}}dx$ *(com algumas alterações, estamos perante* $\int f^{\,p} f'$ *)*

$$\int \frac{1}{\sqrt{(1+x^2)\ln\left(x+\sqrt{1+x^2}\right)}}dx = \int \frac{1}{\sqrt{(1+x^2)}}\frac{1}{\sqrt{\ln\left(x+\sqrt{1+x^2}\right)}}dx$$

$$= \int \overbrace{\frac{1}{\sqrt{(1+x^2)}}}^{f'}\left(\underbrace{\ln\left(x+\sqrt{1+x^2}\right)}_{f}\right)^{\overbrace{\frac{-1}{2}}^{p}}dx = 2\left(\ln\left(x+\sqrt{1+x^2}\right)\right)^{\frac{1}{2}}+C,\quad C\in\Re$$

Cálculos auxiliares:

$$\left(\ln\left(x+\sqrt{1+x^2}\right)\right)' = \frac{\left(x+\sqrt{1+x^2}\right)'}{x+\sqrt{1+x^2}} = \frac{1+\frac{2x}{2\sqrt{1+x^2}}}{x+\sqrt{1+x^2}} = \frac{\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}$$

$$= \frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}} \times \frac{1}{x+\sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}}$$

c) $\int \frac{\cos\left(arcsen\left(\frac{x}{2}\right)\right)}{\sqrt{4-x^2}}dx$ *(com algumas adaptações, estamos perante* $\int \cos f \times f'$*)*

$$\int \frac{\cos\left(arcsen\left(\frac{x}{2}\right)\right)}{\sqrt{4-x^2}}dx = \int \cos\left(\underbrace{arcsen\left(\frac{x}{2}\right)}_{f}\right)\underbrace{\frac{1}{\sqrt{4-x^2}}}_{f'}dx = sen\left(arcsen\left(\frac{x}{2}\right)\right)+C,\quad C\in\Re$$

$$= \frac{x}{2}+C,\quad C\in\Re$$

Cálculos auxiliares:

$$\left(arcsen\left(\frac{x}{2}\right)\right)' = \frac{\left(\frac{x}{2}\right)'}{\sqrt{1-\left(\frac{x}{2}\right)^2}} = \frac{\frac{1}{2}}{\sqrt{\frac{4-x^2}{4}}} = \frac{\frac{1}{2}}{\frac{1}{2}\sqrt{4-x^2}} = \frac{1}{\sqrt{4-x^2}}$$

d) $\int \frac{e^{arctgx}+x\ln\left(1+x^2\right)+1}{1+x^2}dx$

$$\int \frac{e^{arctgx}+x\ln\left(1+x^2\right)+1}{1+x^2}dx = \int e^{\overbrace{arctgx}^{f}}\underbrace{\frac{1}{1+x^2}}_{f'}dx + \int \frac{x}{1+x^2}\ln\left(1+x^2\right)dx + \int \frac{\overbrace{1}^{f'}}{\underbrace{1+x^2}_{1+f^2}}dx$$

$$= e^{arctgx}+c_1+\frac{1}{2}\int \underbrace{\frac{2x}{1+x^2}}_{f'}\underbrace{\left(\ln\left(1+x^2\right)\right)^{\overbrace{1}^{p}}}_{f}dx + arctg(x)+c_2,\quad c_1,c_2\in\Re$$

$$= e^{arctgx}+\frac{1}{2}\frac{\left(\ln\left(1+x^2\right)\right)^2}{2}+arctg(x)+C,\quad C\in\Re$$

$$= e^{arctgx} + \frac{\left(\ln\left(1+x^2\right)\right)^2}{4} + arctg(x) + C, \quad C \in \Re$$

e) $\int \frac{a^{5x}-5}{\sqrt{a^x}} dx$

$$\int \frac{a^{5x}-5}{\sqrt{a^x}} dx = \int \left(a^{5x}-5\right) a^{\frac{-x}{2}} dx = \int \left(a^{5x} a^{\frac{-x}{2}} - 5a^{\frac{-x}{2}} \right) dx = \int a^{5x} a^{\frac{-x}{2}} dx - 5\int a^{\frac{-x}{2}} dx$$

$$= \int a^{\frac{9x}{2}} dx - 5\int a^{\frac{-x}{2}} dx = \frac{2}{9}\int \frac{9}{2} a^{\frac{9x}{2}} dx + 5\times 2\int \left(-\frac{1}{2}\right) a^{-\frac{x}{2}} dx$$

$$= \frac{2}{9}\frac{a^{\frac{9x}{2}}}{\ln a} + 10\frac{a^{\frac{-x}{2}}}{\ln a} + C, \quad C \in \Re$$

$$= \frac{2}{\ln a}\left(\frac{a^{\frac{9x}{2}}}{9} + 5a^{\frac{-x}{2}} \right) + C, \quad C \in \Re$$

f) $\int \frac{x + arcsen(2x)}{\sqrt{1-4x^2}} dx$ *(Com algumas adaptações, estamos perante duas primitivas do tipo* $\int f^p f'$*)*

$$\int \frac{x + arcsen(2x)}{\sqrt{1-4x^2}} dx = \int \frac{x}{\sqrt{1-4x^2}} dx + \int \frac{arcsen(2x)}{\sqrt{1-4x^2}} dx$$

$$= \frac{-1}{8}\int \underbrace{-8x}_{f'} \Big(\underbrace{1-4x^2}_{f}\Big)^{\overbrace{\frac{-1}{2}}^{p}} dx + \frac{1}{2}\int arcsen(2x) \frac{1\times 2}{\sqrt{1-(2x)^2}} dx$$

$$= \frac{-2}{8}\left(1-4x^2\right)^{\frac{1}{2}} + \frac{1}{4}\left(arcsen(2x)\right)^2 + C, \quad C \in \Re$$

$$= \frac{-1}{4}\left[\left(1-4x^2\right)^{\frac{1}{2}} - \left(arcsen(2x)\right)^2 \right] + C, \quad C \in \Re$$

g) $\int \sqrt{\frac{\ln\left(x+\sqrt{1+x^2}\right)}{1+x^2}} dx$ *(Estamos perante* $\int f^p f'$ *)*

$$\int \sqrt{\frac{\ln\left(x+\sqrt{1+x^2}\right)}{1+x^2}}\,dx = \int \left(\underbrace{\ln\left(x+\sqrt{1+x^2}\right)}_{f}\right)^{\overbrace{\frac{1}{2}}^{p}} \underbrace{\frac{1}{\sqrt{1+x^2}}}_{f'}\,dx$$

$$= \frac{2}{3}\left(\ln\left(x+\sqrt{1+x^2}\right)\right)^{\frac{3}{2}} + C, \quad C \in \Re$$

Teste Sobre Primitivação Imediata

1. Resolva as primitivas abaixo

a) $\int \left(\frac{\pi}{4} + \frac{e^x}{\sqrt{1-e^x}} \right) dx$

b) $\int \frac{4}{4+x^2} dx$

c) $\int \frac{e^{x+1}}{1+e^x} dx$

d) $\int \frac{e^x}{\sqrt{4-e^{2x}}} dx$

e) $\int \frac{3sen\ x}{1+\cos^2(x)} dx$

f) $\int \frac{\ln(x)}{x(1+\ln^2 x)} dx$

g) $\int \frac{2^x}{4^x+1} dx$

2. Calcule $\int \frac{1}{\sqrt{x}(1-\sqrt{x})} dx$**, e verifique o seu resultado usando a definição de primitiva de uma função**.

3. Determine a função $f(x)$ **tal que :**

$$f(0)=1$$
$$f'(0)=2$$
$$f''(x)=x^2+x+2$$

4. Diga, justificando, se existe alguma função tal que :

$$f'(x)=x^2+1$$
$$f(0)=1$$
$$f(1)=5$$

PROPOSTA DE RESOLUÇÃO

1a) $\int \left(\frac{\pi}{4}+\frac{e^x}{\sqrt{1-e^x}}\right) dx = \int \frac{\pi}{4}dx + \int \frac{e^x}{\sqrt{1-e^x}}dx$

$= \frac{\pi}{4}x - 2\left(1-e^x\right)^{\frac{1}{2}} + C,\quad C \in \Re$

b) $\int \frac{4}{4+x^2}\ dx = 4\int \frac{1}{4\left(1+\left(\frac{x}{2}\right)^2\right)}dx$

$= 2\int \frac{\frac{1}{2}}{\left(1+\left(\frac{x}{2}\right)^2\right)}dx$

$= 2arctg\left(\frac{x}{2}\right) + C$

c) $\int \frac{e^{x+1}}{1+e^x}\ dx = \int \frac{ee^x}{1+e^x}dx$

$= e\int \frac{e^x}{1+e^x}dx$

$= eln|1+e^x| + C,\quad C \in \Re$

d) $\int \frac{e^x}{\sqrt{4-e^{2x}}}\ dx = \int \frac{e^x}{\sqrt{4\left(1-\left(\frac{e^x}{2}\right)^2\right)}}dx$

$= \frac{1}{2}\int \frac{e^x}{\sqrt{\left(1-\left(\frac{e^x}{2}\right)^2\right)}}dx$

$$=\frac{1}{2}.2\int \frac{\frac{1}{2}e^{x}}{\sqrt{\left(1-\left(\frac{e^{x}}{2}\right)^{2}\right)}}dx$$

$$= arcsen\left(\frac{e^{x}}{2}\right)+C,\quad C\in\Re$$

e) $\int \frac{3sen\ x}{1+\cos^{2}(x)}\ dx = -3\int \frac{-sen(x)}{1+\cos^{2}(x)}dx$

$$= -3\ arctg(\cos x)+C,\quad C\in\Re$$

f) $\int \frac{\ln(x)}{x(1+\ln^{2}x)}\ dx = \int \frac{\frac{\ln x}{x}}{1+\ln^{2}(x)}dx$

$$= \frac{1}{2}\int \frac{2\ln x\frac{1}{x}}{1+\ln^{2}(x)}dx$$

$$= \frac{1}{2}\ln\left|1+\ln^{2}(x)\right|+C,\quad C\in\Re$$

g) $\int \frac{2^{x}}{4^{x}+1}dx = \int \frac{2^{x}}{1+\Big(\underbrace{2^{x}}_{f}\Big)^{2}}dx$

$$= \frac{1}{\ln 2}\int \frac{\overbrace{2^{x}\ln 2}^{f'}}{1+\Big(\underbrace{2^{x}}_{f}\Big)^{2}}dx$$

$$= \frac{1}{\ln 2}arctg(2^{x})+C,\quad C\in\Re$$

2. $\int \frac{1}{\sqrt{x}\left(1-\sqrt{x}\right)}dx = \int \frac{\frac{1}{\sqrt{x}}}{\left(1-\sqrt{x}\right)}dx$ *(Com algumas transformações estamos perante* $\int \frac{f'}{f}$ *)*

Vamos multiplicar e dividir por 2 a primitiva, obtendo:

$$= \quad -2\int \frac{\overbrace{\frac{-1}{2}\frac{1}{\sqrt{x}}}^{f'}}{\underbrace{\left(1-\sqrt{x}\right)}_{f}}dx$$

$$= -2\ln\left|1-\sqrt{x}\right| + C, \quad C \in \Re$$

Vamos agora verificar se $\int \frac{1}{\sqrt{x}\left(1-\sqrt{x}\right)}dx = -2\ln\left|1-\sqrt{x}\right| + C, \quad C \in \Re$. *Caso seja verdade, então, por definição de primitiva de uma função,* $\left(-2\ln\left|1-\sqrt{x}\right| + C\right)' = \frac{1}{\sqrt{x}\left(1-\sqrt{x}\right)}$.

$$\left(-2\ln\left|1-\sqrt{x}\right| + C\right)' = -2\frac{\frac{-1}{2\sqrt{x}}}{1-\sqrt{x}} + 0$$

$$= \frac{\frac{1}{\sqrt{x}}}{1-\sqrt{x}} = \frac{1}{\sqrt{x}\left(1-\sqrt{x}\right)}$$

Fica assim provado que $\int \frac{1}{\sqrt{x}\left(1-\sqrt{x}\right)}dx = -2\ln\left|1-\sqrt{x}\right| + C, \quad C \in \Re$.

3. $f'(x) = \int \left(x^2 + x + 2\right)dx = \frac{x^3}{3} + \frac{x^2}{2} + 2x + C, C \in \Re$

Como $f'(0) = 2$*, temos:*

$2 = 0 + 0 + 0 + C \Rightarrow C = 2$, *logo* $f'(x) = \frac{x^3}{3} + \frac{x^2}{2} + 2x + 2$. *Teremos então,*

$$f(x) = \int \left(\frac{x^3}{3} + \frac{x^2}{2} + 2x + 2 \right) dx = \frac{x^4}{12} + \frac{x^3}{6} + x^2 + 2x + C, \ C \in \Re.$$

Como $f(0) = 1$, *temos*

$1 = 0 + 0 + 0 + 0 + C \Rightarrow C = 1$, *logo*

$$f(x) = \frac{x^4}{12} + \frac{x^3}{6} + x^2 + 2x + 1$$

4. $f(x) = \int (x^2 + 1) dx = \frac{x^3}{3} + x + C$

Como $f(0) = 1$, *temos* $1 = 0 + 0 + C$, *logo* $1 = C$.

Por outro lado dizem-nos que $f(1) = 5$, *logo* $5 = \frac{1}{3} + 1 + C$, *logo* $\frac{11}{3} = C$.

Ora, como C não pode tomar simultaneamente dois valores diferentes, não existe nenhuma função que obedeça às duas condições impostas.